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An inconsistency in the simulation of Bose-Einstein correlations

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Abstract. We show that the formalism commonly used to implement Bose-Einstein correlations in Monte-Carlo simulations can lead to values of the two-particle correlator significantly smaller than unity, in the case of sources with strong position-momentum correlations. This is more pronounced when the phase space of the emitted particles is strongly reduced by experimental acceptance or kinematic analysis selections. It is inconsistent with general principles from the coherent state formalism according to which the Bose-Einstein correlator is larger than unity. This inconsistency seems to be rooted in the fact that quantum mechanical localization properties are not taken into account properly.

1 Introduction

Hanbury-Brown Twiss interferometry has been used [1,2] in both high energy and nuclear physics to determine the space-time dimensions of the emitting source created during nuclear collisions by using the effect of the interference pattern between two identical produced bosons [3–5].

The source parameters derived from fits to the correlation function are difficult to interpret directly as real geometric quantities, being sensitive to the transverse and longitudinal dynamical expansion of the system [6–11], which result in a momentum dependence of the extracted source radii [12–14], to long lived resonance decays [15–19], the Coulomb interaction [20,21], and final state rescattering [22]. There has been recent experimental evidence for flow effects [23,11] and one possible implication of flow is that distant points of emission in the source volume cannot emit particles with closely differing momenta, and thus do not contribute to the small relative pair momentum region [24]. It is also anticipated that strong absorption must exist in the case of large stopping power; a particle originating at the side of the source opposite to the direction of its momentum cannot easily propagate through the source to be seen by the detector, and therefore only a limited region of the source will be seen, noted already in AGS studies [25]. It is thus interesting to try to further probe the relationship between source geometry, dynamical expansion, and kinematical regimes viewed by the measuring apparatus.

To study the effect of position-momentum correlations on the shape of the correlation functions, a simple Monte Carlo phase-space model controlled by a few macroscopic parameters was developed. As is the case with more detailed and sophisticated microscopic event generators, there is no Bose-Einstein symmetrization effect included from first principles [26,27,22]. The Bose-Einstein correlations were then added to the initial distributions by including the symmetrization in the form of a weight calculated for each pair of identical particles, a procedure found extensively in the literature [28,16,22,29].

In the present work we show that there are limitations to this formalism, as it is an approximation and several assumptions are implicit in its derivation. This will be demonstrated on the basis of the Monte-Carlo calculations. We briefly check that our model produces results consistent with theoretical predictions as well as with other simulation studies in case of similar phase space distributions. In our further studies of source models with position momentum dependence, we then find that the current praxis of including Bose-Einstein correlations leads to results quantitatively inconsistent with general principles. Here we restrict our discussion to pointing to a possible origin of this problem.

This paper proceeds as follows: Section 2 describes the Monte-Carlo model and the inclusion of the Bose-Einstein effect, Sect. 3 presents the model results and in Sect. 4, we turn to the discussion of the observed problems.

2 The model

Our model produces constant multiplicity events from an analytically given phase space distribution by a Monte Carlo technique. The model provides the phase space distribution of particles at the points (x^μ, p^μ) of their last interaction, with no assumptions about the dynamical evolution of the collision. The collision region is described in terms of a few macroscopic parameters defining the spatiotemporal extension of the source, such as the source shape and size, and the dynamical features of the system, such as temperature and collective flow. All the particles are assumed to be pions, and resonances are not included.

As for all existing event generators, the obtained phase space distribution does not contain Bose-Einstein correlations. To include the latter, we applied the following, widely used prescription [20,31]: Each identical pion pair emitted from the points $(\mathbf{r_i}, t_i)$ and $(\mathbf{r_j}, t_j)$ is weighted with the Born probability of a symmetrized two-pion plane wave,

$$| \Psi_{ij} |^2 = 1 + cos[(\mathbf{r_i} - \mathbf{r_j}) \cdot (\mathbf{p_i} - \mathbf{p_j}) - (t_i - t_j)(E_i - E_j)] \quad (1)$$

and the correlator is defined as the normalized sum over all different pairs $| \Psi_{ij} |^2$. At least two approximations are made here. Firstly, higher order symmetrization effects which exist for any state of more than 2 identical pions are neglected: the probability corresponding to a proper N-particle symmetrized wave function is approximated by a sum over the pair probabilities $| \Psi_{ij} |^2$. The corresponding approximation error is known to decrease with decreasing pion density or increasing source size [40] and the pion source created in heavy ion collisions is usually assumed to be sufficiently large for this approximation to be good. Secondly, applying (1) violates strict energy momentum conservation since it increases the probability of finding pairs at smaller relative momentum, without changing the event multiplicity. In heavy ion collisions, however, the region of relative pair momentum affected by the symmetrization is small compared to the overall region occupied by data. Hence the errors resulting from the neglect of energy momentum conservation are expected to be small [41]. This is in sharp contrast to Bose-Einstein algorithms used in modelling e^+e^- physics [39,42], where energy momentum conservation is implemented correctly but the role of the spatiotemporal distance of particle emission points for the Bose-Einstein weights is neglected. In this work, we follow common practice in using (1). The problems we observe with this prescription are, as we shall see, of a different nature and cannot be traced back to the approximations just described.

Our arguments are illustrated in a source model in which emission of on-shell pions at freeze-out is assumed to be instantaneous (there is no time evolution).

$$| \Psi |^2 = 1 + cos[(\mathbf{r_1} - \mathbf{r_2}) \cdot (\mathbf{p_1} - \mathbf{p_2})] \quad (2)$$

The correlator defined via (2) is a function of the three-dimensional relative momentum component $\mathbf{q}$. The correlator $C(\mathbf{q})$ is then obtained for each bin as the sum over pion pairs weighted by $| \Psi |^2$ and normalized to the sum of unweighted pion pairs, cf. [20]. The Coulomb interaction between the pions is not simulated.

Here, we streamline our presentation by restricting it to the results of the one-dimensional fits of $C(q_3)$ in terms of the 3-momentum difference q_3

$$C(q_3) = 1 + \lambda \exp[-q_3^2 R_3^2] \qquad q_3 = | \mathbf{p_1} - \mathbf{p_2} | \ . \quad (3)$$

All calculations are done in the center of mass system.

3 Model results

In contrast to full event generators like Venus, RQMD, ARC, etc., which try to incorporate all the physics expected to be present in a heavy ion collision, the purpose of our model was to isolate and study one important effect: the geometrical and dynamical interpretation of HBT parameters in the presence of radial flow and realistic experimental acceptance. The model's simplicity allowed the well controlled study of a wide set of different flow and acceptance conditions. Thus in the course of this study, we have found inadequacies in the common practice (1) of including Bose-Einstein correlations which become particularly apparent for sources with strong position-momentum dependence where certain kinematical selections are imposed. Our presentation will first illustrate this effect in an instructive way for the extreme case of complete position-momentum correlations in the source. Next, we will summarize the results of flow and acceptance effects on the HBT parameters for realistic phase space distributions and acceptance criteria.

3.1 Source with complete position-momentum correlations

Here, we calculate the correlation function for a linear source in the beam (z) direction with neither transverse spatial extension nor transverse momentum dependence. The longitudinal momentum is chosen to be completely due to flow,

$$p(z) = D\,z\,, \quad (4)$$

where p has units of GeV/c and z is in fermi, D being a constant. This distribution represents a source expanding in the z-direction for which the argument in the cosine of (2) reduces to $(\mathbf{r_1} - \mathbf{r_2}) \cdot (\mathbf{p_1} - \mathbf{p_2}) = Q_3^2/D$. For $D = 0.02$, this leads to the correlation function shown in Fig. 1. The source in (4) has a total position-momentum correlation and in this case, as can be checked analytically [37], the correlator obtained for (4) with the *cos*-prescription oscillates between 2 and 0. Introducing a Gaussian spatial smearing of the emission points in (4), as might be motivated by the picture of a limited quantum mechanical localization of particles, one sees that the oscillations of the correlator decrease. Still, the correlator can drop below unity, and this is in strong contradiction to calculations from first principles [36] which ascertain that for arbitrary sources the correlator is always larger than unity. We next investigate in how far this behaviour persists for more realistic phase space distributions.

3.2 Position-momentum dependence in a realistic model

To study more realistic scenarios, we generate pions according to macroscopic model parameters that correspond to an instantaneous Gaussian source with realistic momentum distributions. No attempt is made either to reproduce any data or to create a fully realistic Monte-Carlo

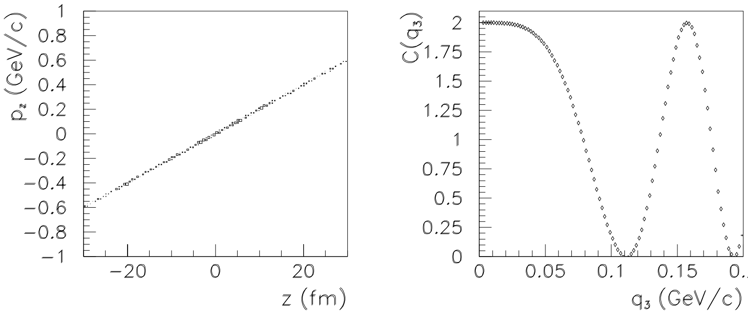

Fig. 1. For a linear source expanding in the z direction according to (4), $D = 0.02$. *Left*: Momentum p_z in the beam direction as a function of the z position of emitted pions. *Right*: The resulting two-pion correlation in q_3 using the formula in (2)

event generator. Accordingly the spatiotemporal part of the source is modelled by

$$G = \text{const.} \times \exp\left[-(x^2 + y^2 + z^2)/R^2\right]. \qquad (5)$$

The transverse momentum dependence and the rapidity dependence chosen for the case of no flow (i.e., no position momentum dependence in the source) are

$$dN/dp_T = A\, p_T \exp\left(-p_T/B\right),$$
$$y = c\sqrt{-2\ln a}(cos(2\pi b)). \qquad (6)$$

As an input parameter, we use a Gaussian radius $R = 6$ fm, motivated by the hard sphere radius of the ^{208}Pb incoming projectile. The input for the momentum distributions is chosen according to the measured transverse momentum (p_T) and rapidity (y) distributions in the CERN 158 GeV/n Pb+Pb data [30]. The inverse slope parameter of the p_T-distribution is taken to be $B = 200$ MeV and A is an arbitrary normalization constant. The rapidity ($y \equiv \frac{1}{2}\log\frac{E+p_z}{E-p_z}$) dependence is specified by random numbers a and b which are uniformly distributed in the interval (0,1), c being a constant.

To incorporate radial flow in the model, we modify the phase space distribution (Eqs. 5, 6) by introducing a radial flow $\beta(r)$ of the emission points,

$$\beta(r) = 1 - e^{-r/f}, \qquad (7)$$

where f is an adjustable parameter. For different flow strengths f, the radial dependence is shown in Fig. 2. Superimposed is the mean value of the radial velocity

$$\beta_r = \frac{\mathbf{p}\cdot\mathbf{r}}{E \mid r \mid}, \qquad (8)$$

extracted from simulated Pb+Pb Venus events, version 4.12 [26]. Here E denotes the total energy of the pion and r its radial distance from the source center. One sees that a choice of $f = 9$ fm fits the Venus data very well. This value of the flow strength, $f = 9$, was used for simulating a "realistic" flow. No attempt was made to reproduce any other Venus distribution. As demonstrated in [12], the flow dependence of the observables is mainly due to the size of the flow, while its functional shape plays a somewhat secondary role. The order of magnitude of the flow velocity extracted here ($\sim 0.35c$) can be compared to the flow parameter η_f in [12].

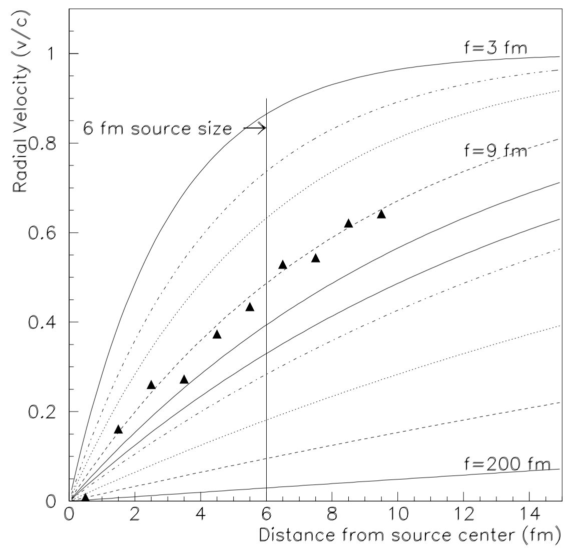

Fig. 2. The radial flow velocity as a function of the radial distance from the center of a pure Gaussian source parametrized according to (3). The flow extracted from Venus (v. 4.12) is well represented by this parametrization with f=9 fm (*filled triangles*). The *vertical line* shows the true 6 fm Gaussian source size

We have generated events for different values of the flow strength f and the result was verified to be independent of statistics. Each Monte-Carlo event contained typically 100 identical pions. For the relative pair momentum differences, 5 MeV bins were used and the results were checked to be insensitive to bin sizes in the range of 5-20 MeV. The original aim was the study of the flow and acceptance dependence of HBT radius parameters. Indeed, our simple model shows reasonable physical properties. Especially the flow dependence of the 1- and 3-dimensional k_T integrated HBT radius parameters is in qualitative agreement with that obtained in other model studies [35,12,29]: The HBT radii decrease with increasing flow strength f, since the effective emission region ("region of homogeneity" [10]) for pion pairs with small relative momenta decreases for increasing f. Also, we have considered the so far little studied effect of the detector acceptance on the HBT radius parameters. In Fig. 3 (left), we show for the case of a realistic flow strength $f = 9$ fm the freezeout positions of all generated pions and of

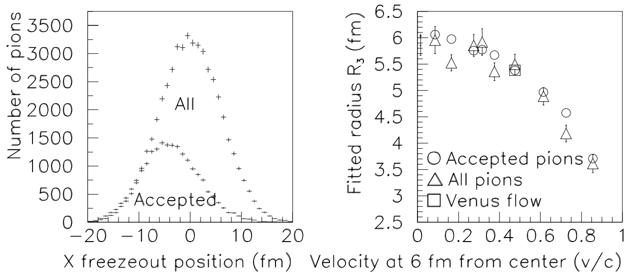

Fig. 3. *Left*: Freezeout positions in the x direction for all pions and those satisfying a kinematical acceptance cut, where the initial pion momenta have been changed by adding radial flow extracted from Venus (eg. (3) with f=9 fm)). *Right*: The HBT radius R_3 as a function of the flow velocity at a distance of r=6 fm from the source center for the radial flow profiles in Fig. 2

those satisfying the (p_T, y) acceptance criteria of a typical magnetic spectrometer [34]. The emission region of the detected pions is clearly smaller than the total emission region. The corresponding HBT radius parameter R_3 is presented in Fig. 3 (right) as a function of the flow strength f for the cases with a realistic magnetic spectrometer acceptance and without. In both cases, the HBT radii decrease with increasing flow and the acceptance dependence is very small. From all this we conclude here only that our model is not too oversimplified and reproduces essential features obtained in more complete simulations.

We now consider a simple modification of our model, the introduction of an absorption cut. To this aim, we impose on the Monte Carlo output, defined by (5-8), a kinematical cut which effectively strengthens the position-momentum correlation in the source: pions are only emitted, if their momentum vector is in an angle less than 45 degrees around the direction of their position vector, i.e., each pion has to move out of the source in a 45 degree cone. Under these conditions, the correlation functions in q_3 displayed in Fig. 4 have been obtained, where, additionally, cuts in single particle transverse momentum of 0.1 GeV/c - 0.3 GeV/c have been applied.

We emphasize that in contrast to the pathological source discussed in Sect. 3.1, this modified source shows a rather reasonable phase space distribution which is difficult to reject a priori. Still, the correlator obtained from the cos-prescription (1) again drops below unity. Especially, when p_T is restricted to small values, the dip in the correlation function around 0.08 GeV/c figures more prominently. By construction of the weights in this Monte-Carlo, the tail of the correlator is normalized to unity, so this dip is not associated with the overall normalization of the correlation function, which is the same in all cases depicted in Fig. 4. Such a characteristic dip can be produced in $C(q_3)$ given at least a p_T cut, and *either* radial flow *or* the angle cut. We thus conclude that it arises when imposing an acceptance cut on the pions emitted from the Gaussian model with strong position-momentum correlation. It is worth noting that the half-width of the correlation function remains nearly identical in all four cases in Fig. 4.

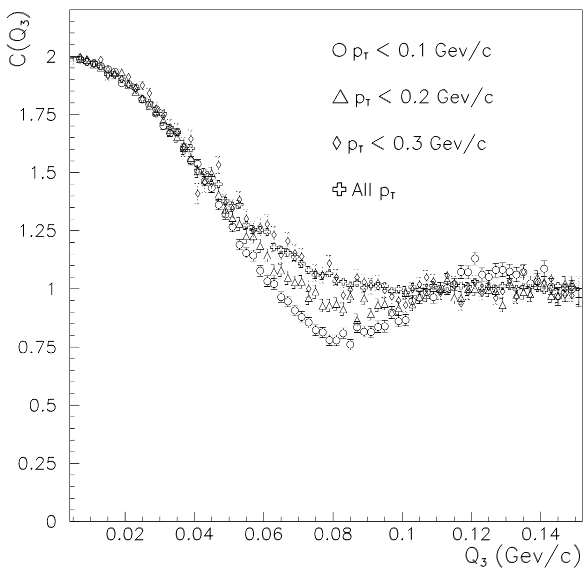

Fig. 4. Pion correlation in q_3, with a horizon cut imposed such that the pion momentum vector must be within a 45 degree angle of the radial vector, for four different selections in single pion transverse momentum

4 Discussion

Existing event generators do not propagate (anti)-symmetrized wave functions and hence face a conceptual difficulty in incorporating the effect of Bose-Einstein correlations [38]. The current practice of modifying the weight of pion pairs by the Born probability $|\Psi|^2$ of symmetrized plane waves does not address this problem properly. Here, we have shown for the first time that this conceptual difficulty can have significant quantitative consequences. Especially, the dip observed in the correlator Fig. 4 shows that the formalism leading to (2) contradicts the coherent state formalism arising from quantum field theory, using [36]

$$C(\mathbf{q},\mathbf{K}) = 1 + \frac{\left|\int d^4x\, S(x,K)\, e^{iq\cdot x}\right|^2}{\int d^4x\, S(x,K+\frac{1}{2}q) \int d^4x\, S(x,K-\frac{1}{2}q)} \tag{9}$$

with $q = p_1 - p_2$ and $K = (p_1 + p_2)/2$. Equation (9) does not contain finite multiplicity corrections which are of order $O(\frac{1}{N})$, and its derivation assumes emission from a heat bath and hence energy-momentum conservation is neglected. Based on these assumptions, it is derived without further approximation. Both schemes, (9) and (2), normalize the correlator by requiring that in the limit of large relative momentum, $C(\mathbf{q},\mathbf{K})$ approaches unity. However, clearly, the correlation function (9) cannot become smaller than unity while Fig. 4 does. The observed dip cannot be traced back to a normalization problem, to the neglect of energy momentum conservation, or to dropping higher order symmetrizations, thus its origin must be different. The approximate method using (2) is based on a semiclassical picture for a set of discrete space-time points. It produces effects that are not always consistent with (9) and which can become non-negligible as seen in Fig. 4. They are more pronounced in the presence of strong position-momentum dependence when the long-range characteristics of the argument $(\mathbf{r_1}-\mathbf{r_2})\cdot(\mathbf{p_1}-\mathbf{p_2})$ in (2) play an important role.

The discrepancy in methodology can be even more drastic; an analytical calculation based on (9) using the source given in (4) does not show these oscillations [37], while using (2) with the same source yields Fig. 1. However, (2) is at this time the only method available to build correlation functions from the space-time output of microscopic event generators [28,16,22,31,29]. This formalism has yielded reasonable results under the less severe conditions studied until now. The exact expression given in (9) however cannot be used in a Monte-Carlo simulation when the Monte-Carlo event generator does not provide the source density function of the mean momentum $\mathbf{K}$. The probabilistic Monte-Carlo approach does not allow to deal with the quantum mechanics effects involved here. Without a solution to this problem, since the procedure based on (2) has been used by many groups in the recent literature [28,16,22,29], it is now henceforth clearly important to improve the current Monte-Carlo Bose-Einstein formalism and to estimate the errors involved in the procedure using (2).

5 Conclusions

We have shown that the cos-prescription commonly used to include Bose-Einstein effects in the Monte-Carlo simulation can lead to results not consistent with first principles. This calls into question its quantitative and qualitative reliability, especially for the case that certain kinematical selections are applied when strong position-momentum correlations are inherent in the pion source. A deeper understanding of this simulation formalism is necessary in order to make more detailed analyses of dynamical issues and acceptance effects. Most remarkably, the cos-prescription (1) interprets both position $\mathbf{r}_i$, t_i, and momenta $\mathbf{p}_i$ returned by an event generator as sharp (classical) phase space coordinates which violates the Heisenberg uncertainty principle. On the other hand, a smearing of the emission points, motivated e.g. by the picture of a limited quantum mechanical particle localization, allows to remedy the unphysical dip in the correlator at least partly, see Section 3.1. This observation may indicate in our opinion that the inconsistencies of the prescription (1) presented here are rooted in an incorrect treatment of the quantum mechanical particle localization. This points out the need for an advanced quantum mechanical Monte-Carlo event generator that can properly describe Bose-Einstein correlation functions.

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